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EFFECT OF TURBULENCE OF THE OUTER STREAM ON THE TRANSITION FOR SOME CLASSES OF SELF-SIMILAR FLOWS

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It is shown that in the presence of very low levels of pulsations at the outer limit of the laminar boundary layer the energy of neutral oscillations within the layer itself reaches very high values. This predetermines the transition to turbulent flow.

1. Primary Motion and Equation of the Oscillations

For the study of the stability of laminar flow, as is usually done in mechanics in the theory of stability, we will examine the primary motion on which the perturbed motion, caused by the presence of disturbances in the form of a degree of turbulence E_0 of the outer stream, is superimposed. In the general case one can assume that the perturbed motion affects the primary motion. The equations of the plane motion of a viscous incompressible fluid, written through the stream function $\psi(x, y, t)$, have the form [1]

$$\frac{\partial}{\partial t}(\Delta\psi)-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\Delta\psi)+\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\Delta\psi)=\nu\Delta\Delta\psi. \tag{1}$$

The stream function $\psi(x, y, t)$ describes the instantaneous state of the liquid or gas. We designate the primary motion through $\Psi(x, y)$, while the stream function $\psi'(x, y, t)$ will describe the perturbed motion. Then $\psi = \Psi + \psi'$, where the stream function ψ' can be determined from the equation

$$(\Delta\psi')_t+\Psi_y(\Delta\psi')_x+\psi'_y(\Delta\Psi)_x-\Psi_x(\Delta\psi')_y-\psi'_x(\Delta\Psi)_y=\nu\Delta\Delta\psi'. \tag{2}$$

Considering the motion in the boundary layer, by substituting $\psi = \Psi + \psi'$ into Eq. (1) and carrying out the averaging (in the ergodic sense) of the equation obtained we arrive at the average equation of the primary motion in the form [2]

$$uu_x+vu_y+\overline{u'^2_x}+\overline{u'v'_y}=u_eu_{ex}+\nu u_{yy}, \tag{3}$$

where

$$u=\Psi_y;\quad v=-\Psi_x;\quad u'=\psi'_y;\quad v'=-\psi'_x.$$

The last two terms on the left side of (3) characterize the effect of the perturbed motion on the primary motion.

Henceforth, solutions for system (3) like those which follow flow of the Folkner — Skan type, i.e., $u_e = cx^m$, where the index e pertains to the outer nonviscous stream, will be examined as solutions of the equations

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of the primary motion. In place of $\Psi(x, y)$ we introduce the function $F(\eta)$ such that

$$\Psi(x, y) = F(\eta)\, \nu\, [\xi (2-\beta)]^{0.5}, \tag{4}$$

where

$$F_\eta = \frac{u}{u_e}; \quad \eta = \frac{y}{x}\left[\frac{\xi}{2-\beta}\right]^{0,5}; \quad \xi = \frac{u_e x}{\nu}; \quad \beta = \frac{2m}{m+1};$$

in place of $\psi'(x, y, t)$ we introduce the function $F'(\eta)$, determined by the relation*

$$\psi'(x, y, t) = F'(\eta)\, \nu\, [\xi (2-\beta)]^{0.5}\, \Phi'(t). \tag{5}$$

Let us substitute (4) and (5) into Eq. (2), then multiply both sides of the equation obtained by $\Phi'(t)$, and average all the terms of the resulting equation with respect to time. We then obtain the following linear equation:

$$a_4 F'_{\eta\eta\eta\eta} + a_3 F'_{\eta\eta\eta} + a_2 F'_{\eta\eta} + a_1 F'_\eta + a_0 F' = 0, \tag{6}$$

where

$$a_0 = -4\lambda\xi (m^2-1) + 2\eta^2 F_{\eta\eta\eta} (m-1)^2 (m+1) + 2\eta F_{\eta\eta} (m^2-1)(5m-3) +$$
$$+ 6F_\eta (m-1)(m+1)^2 + 4\xi F_{\eta\eta\eta} (m+1)^2 + \frac{1}{\xi}(m-5)(m-3)(m^2-1);$$
$$a_1 = 2\eta^2 F_{\eta\eta} (3-m)(m-1)^2 + 4\eta F_\eta (3-m)(m-1)(3m-1) -$$
$$- 4\eta\lambda\xi (3m-1)(m-1) + \frac{\eta}{\xi}(m-3)(m-1)(15m^2-28m+5) +$$
$$+ 6F(m+1)(m^2-1) - 4\xi F_{\eta\eta} (m+1)(3m-1);$$
$$a_2 = \eta^2 \left\{\frac{1}{\xi}(m-1)^2 (25m^2-74m+45) + 2F_\eta (3-m)(m-1)^2 -\right.$$
$$\left. - 4\lambda\xi (m-1)^2\right\} + 2\eta F (m^2-1)(5m-3) - 8\lambda\xi^2 (m+1) - 4\xi F_\eta (m+1)(3m-1) + 12(m^2-1)(3m-1);$$
$$a_3 = 2\frac{\eta^3}{\xi}(m-1)^3 (5m-7) + 2\eta^2 F (m-1)^2 (m+1) + 4\eta (m^2-1)(7m-5) + 4\xi F (m+1)^2;$$
$$a_4 = \frac{\eta^4}{\xi}(m-1)^4 + 4\eta^2 (m-1)^2 (m+1) + 4\xi (m+1)^2;$$
$$\lambda = \frac{\nu}{u_e^2}\, \frac{\overline{\Phi' \Phi'_t}}{\overline{\Phi'^2}}.$$

For convenience in the solution of the resulting boundary problem, in the integration of (6) it will be convenient to introduce into the discussion a function $f = F'/F'_\eta(0)$ for problems of the "wake" type and a function $f = F'/F'_{\eta\eta}(0)$ for near-wall problems. Since $\sqrt{\overline{u'^2}}/u_e = F'_\eta \sqrt{\overline{\Phi'^2}}$ in the assignment of the degree of uniform turbulence $E_0 = \sqrt{\overline{u'^2}}/u_e|_{\eta\to\infty}$ of the outer stream one can eliminate from consideration the indeterminate value $\sqrt{\overline{\Phi'^2}} = E_0/F'_\eta(\infty)$ and obtain

$$\frac{\sqrt{\overline{u'^2}}}{u_e} = E_0 \frac{f_\eta}{f_\eta(\infty)}, \quad \frac{\sqrt{\overline{v'^2}}}{u_e} = \{f + (\beta-1)\eta f_\eta\} \frac{E_0 [\xi(2-\beta)]^{-0,5}}{f_\eta(\infty)}. \tag{7}$$

Finally, if we introduce the function $\Psi(x, y)$ in the form (4) into Eq. (3) and use (7), then we obtain Eq. (8), and the resulting system of equations, serving for the determination of the intensity of pulsations in the boundary layer, will have the form

$$F_{\eta\eta\eta} + FF_{\eta\eta} + \beta(1-F_\eta^2) + \left[\frac{E_0}{f_\eta(\infty)}\right]^2 (ff_{\eta\eta} + \beta f_\eta^2) = 0, \tag{8}$$

$$\sum_{i=0}^{i=4} a_i \frac{d^i f}{d\eta^i} = 0. \tag{9}$$

*This form of representation of $\psi'(x, y, t)$ and the hypothesis of a transition site were first pointed out by L. N Shchukin (Moscow) in a report at a seminar on the theory of resistance and heat transfer lead by Professor I. P. Ginzburg, Leningrad State University (1968).

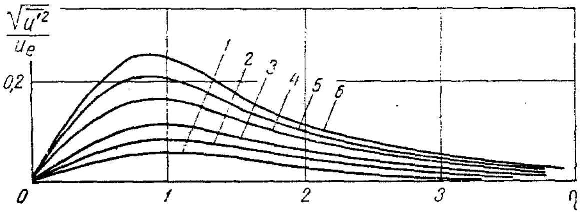

Fig. 1. Law of variation of energy of pulsation motion in a cross section of the boundary layer at a plate with different local Reynolds numbers $\xi = u_e x/\nu(\sqrt{\overline{u'^2}}/u_e \gg \sqrt{\overline{v'^2}}/u_e$ when $0 < y < \delta$): 1) $\xi = 0.5 \cdot 10^6$; 2) $0.1 \cdot 10^7$; 3) $0.2 \cdot 10^7$; 4) $0.4 \cdot 10^7$; 5) $0.7 \cdot 10^7$; 6) 10^7.

2. Analysis of Equations Obtained

Equation (8), according to the assumption made concerning the nature of the outer stream ($u_e = cx^m$), is self-similar if the function f actually depends only on the similarity coordinate η. However, the coefficients of Eq. (9) reveal a clear dependence on the longitudinal coordinate x, since they contain the local Reynolds number $\xi = Re_x$. An analysis of the coefficients a_i shows that with m = 1 (flow in the vicinity of the stagnation point of the stream) these coefficients will equal $a_0 = 16\xi F_{\eta\eta\eta}$, $a_1 = -16\xi F_{\eta\eta}$, $a_2 = -16\lambda\xi^2 - 16\xi F_\eta$, $a_3 = 16\xi F$, and $a_4 = 16\xi$, respectively. For a steady random process at the outer limit of the boundary layer the random value Φ' and its time derivative Φ'_t are independent; consequently, for such processes $\overline{\Phi'\Phi'_t} = 0$ and $\lambda = 0$, which corresponds here to the concept of neutral oscillations. In this case Eq. (9) is reduced to the form

$$f_{\eta\eta\eta\eta} + Ff_{\eta\eta\eta} - F_\eta f_{\eta\eta} - F_{\eta\eta} f_\eta + F_{\eta\eta\eta} f = 0. \tag{10}$$

As seen from (10), the equation of the oscillations is fully self-similar, which corresponds to the assumption made initially concerning the independence of F' (or f) from ξ.

A study of the coefficients a_i for other types of flows ($m \neq 1$) shows that in the case of neutral oscillations ($\lambda = 0$) their values are described by the equations

$$a_0 = 4\xi\left\{(m+1)^2 F_{\eta\eta\eta} + O\left(\frac{1}{\xi}\right)\right\},$$

$$a_1 = 4\xi\left\{-F_{\eta\eta}(m+1)(3m-1) + O\left(\frac{1}{\xi}\right)\right\},$$

$$a_2 = 4\xi\left\{-F_\eta(m+1)(3m-1) + O\left(\frac{1}{\xi}\right)\right\},$$

$$a_3 = 4\xi\left\{F(m+1)^2 + O\left(\frac{1}{\xi}\right)\right\},$$

$$a_4 = 4\xi\left\{(m+1)^2 + O\left(\frac{1}{\xi}\right)\right\}.$$

For large Reynolds numbers Re_x the terms in brackets will practically, with a degree of accuracy $O(1/\xi)$, not depend explicitly on ξ (or x), and after substitution of the coefficients a_i into Eq. (9) the latter also proves to be self-similar.

3. Boundary Conditions

As the boundary conditions imposed on $F(\eta)$ in the solution of system (8)-(9) for near-wall problems one uses the usual conditions of attachment to the wall at $\eta = 0$ and the condition of the asymptotic emergence of the velocity at a fixed value in the outer stream as $\eta \to \infty$; the velocity pulsations at the wall equal zero; at the outer limit of the boundary layer one uses the following conditions of uniformity of the pulsations and the condition of a fixed degree of turbulence E_0:

at $\eta = 0$

$$F = F_w \text{ (or } 0), \quad F_\eta = f = f_\eta = 0, \quad f_{\eta\eta} = 1, \tag{11}$$

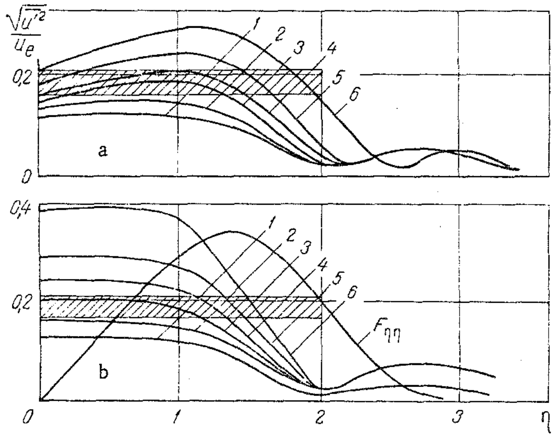

Fig. 2. Law of energy distribution of pulsation motion in a cross section of the wake at the point of division of the stream into circulation and direct flows in the null (a) and first (b) approximations with different degrees of turbulence E_0 of the outer stream. The shaded region corresponds to values obtained from Eqs. (14)-(15) ($E_0 = \sqrt{\overline{u'^2}}/u_e|_{y=\delta}$): 1) $E_0 = 0.00058$; 2) 0.00073; 3) 0.00091; 4) 0.00112; 5) 0.00139; 6) 0.00194.

as $\eta \to \infty$

$$F_\eta \to 1, \quad f_\eta [\xi (2-\beta)]^{0.5} \to f + (\beta - 1)\eta f_\eta.$$

In the analysis of problems of the "wake" type one uses the conditions of symmetry for the parameters of the primary flow and the pulsations at $\eta = 0$; the conditions indicated above are used at the outer limit as $\eta \to \infty$:

at $\eta = 0$

$$F = F_{\eta\eta} = f = f_{\eta\eta} = 0, \quad f_\eta = 1, \tag{12}$$

as $\eta \to \infty$

$$F_\eta \to 1, \quad f_\eta [\xi (2-\beta)]^{0.5} \to f + (\beta - 1)\eta f_\eta.$$

Equations (8) and (9) are solved jointly; when allowance is made for the effect of the perturbed motion on the primary motion the last term in Eq. (8) is retained; otherwise this term is omitted and the ordinary Folkner — Skan equation is used as the equation of the primary motion:

$$F_{\eta\eta\eta} + FF_{\eta\eta} + \beta(1 - F_\eta^2) = 0.$$

4. Results of the Solution

The calculations were carried out for three characteristic types of flows in the boundary layer: for a plate ($\beta = m = 0$), for a separation point at the surface of the body ($F_{\eta\eta} = 0$), and for a point of division of the stream in the wake, which is a full analog of the point of separation at the surface of a body.

The difference between the latter two types of flow will consist entirely in the fact that the pulsations at the wall (when $\eta = 0$) equal zero for near-wall flows, while for flows of the wake type pulsations always occur at the axis (when $\eta = 0$). The calculations were carried out in the null approximation (without allowance for the effect of the perturbed motion on the primary motion) and in the first approximation (with allowance for this effect). Calculations on the determination of the intensity of pulsations at a plate ($\beta = 0$) in the presence of a very low degree of turbulence $E_0 = 0.0002$ of the outer stream are presented in Fig. 1 as an example; such a degree of turbulence was observed in the well-known experiments of Dryden [3]. It is clearly seen how the energy of the pulsations within the boundary layer grows with an increase in the local Reynolds number ξ. The effect of the perturbed motion on the primary motion is absent from the calculations presented in Fig. 1. As the calculations showed, for near-wall problems with small values of E_0 the allowance for the perturbed motion in the equations of the primary motion proves to have a comparatively weak effect on the behavior of the pulsation motion in the viscous near-wall layer. The law of distribution of the pulsations at the point of division of the wake ($F = F_\eta = F_{\eta\eta} = 0$ when $\eta = 0$) in the null and

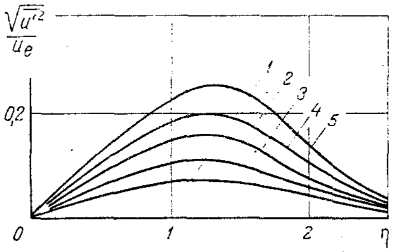

Fig. 3

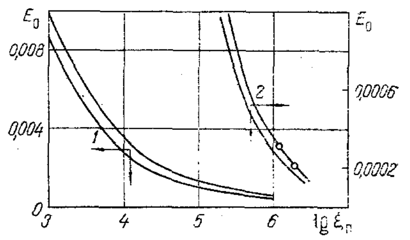

Fig. 4

Fig. 3. Law of energy distribution of pulsation motion in the cross section corresponding to the separation point at the wall with different E_0 in the null and first approximations: 1) E = 0.000438; 2) 0.000683; 3) 0.000945; 4) 0.00125; 5) 0.00166.

Fig. 4. Transitional Reynolds numbers as a function of turbulence E_0 of outer stream for trailing critical point of a wake (region 1) and at a plate (2). Points: Dryden's experiment [3].

first approximations for $\xi = 10^5$ and different E_0 is presented in Fig. 2.

From a comparison of Figs. 2, a and b, it is seen that the allowance for the effect of the perturbed motion on the primary motion somewhat increases the maximum of the pulsations and shifts it toward the axis of the wake. The considerable amplitude of the oscillations within the layer when the degree of turbulence E_0 of the outer stream is very low attracts attention.

The distribution law of the primary pulsation component $\sqrt{\overline{u'^2}}/u_e$ across the viscous layer at the separation point on the surface of the body, with the same value $\xi = 10^5$ as in the wake in the null approximation, is shown in Fig. 3.

As the calculations showed, the solution of the problem in the first approximation does not introduce significant refinements into the solution in the null approximation. The intensity maximum of the pulsations in the near-wall flow (at a plate and at the separation point) is located at a distance of about a quarter of the thickness of the boundary layer from the wall; this maximum approaches the wall with an increase in ξ.

5. Pulsation Intensity for Developed Turbulent Motion

For an estimate of the pulsation intensity of the developed turbulent motion we can use the correlation between the Reynolds stresses $-\rho\overline{u'v'}$ and the total turbulent pulsation intensity $\overline{q^2} = \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$

$$-\overline{u'v'} = \pm k_1 \overline{q^2}. \tag{13}$$

Here the plus sign is taken if $(\partial u/\partial y) > 0$ and vice versa. In this connection, Schubauer and Kleban [4] have observed a distribution of a similar nature, not only near the channel wall but also in the outer region, having much in common with jet flows. In the outer region, where a maximum in $\overline{q^2}$ exists, the proportionality constant k_1 reaches a value of $k_1 \simeq 0.4$ [4]; Townsend [5] indicates a similar value for turbulent jets and wakes. To determine the maximum energy of the turbulent pulsations one must indicate the maximum value of the Reynolds stress $\tau_{max} = (-\rho\overline{u'v'})_{max}$. Within the framework of the present analysis we can use the approximate relations of semiempirical turbulence theory: $\tau_t = \rho\varepsilon_t(\partial u/\partial y)$; here one can examine various models of turbulent viscosity both for the near-wall problems and for problems of the "wake" type. The Prandtl equation $\varepsilon_t = \varkappa\delta(u_e - u_m)$ is the most widely used in the theory of jets and wakes, and the equation $\varepsilon_t = (1/2\sigma^2)x(u_e - u_m)$, where σ is a similarity constant determined by experiment, is also widely used. The profile of the average longitudinal velocity component can also be assigned by various means. Let us consider the following means of assignment [6] of the velocity profile ($\eta = y/\delta$, δ is the width of the layer or jet, and $\Delta u = u - u_e/u_e - u_m$):

$$\Delta u = (1 - \eta^{\frac{3}{2}})^2, \tag{I}$$

(the profile of H. Schlichting)

$$\Delta u = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4, \tag{II}$$

$$\Delta u = 1 - 3\eta^2 + 2\eta^3 \tag{III}$$

(II and III are profiles of A. S. Ginevskii),

$$\Delta u = \frac{1}{2}(1+\operatorname{erf}\eta). \tag{IV}$$

The latter expression for the velocity profile corresponds to a turbulent viscosity model of the type $\varepsilon_t = (1/2\sigma^2)x(u_e - u_m)$ while the coordinate η is the similarity coordinate $\eta = \sigma(y/x)$. Using the condition $\max(k_1\overline{q^2}) = \max[\varepsilon_t(\partial u/\partial y)]$, we obtain the following respective values of the total intensity maximum for the indicated velocity families I-IV: with $\varepsilon_t = \varkappa\delta(u_e - u_m)$

$$\left\{\frac{\sqrt{\overline{q^2}}}{u_e - u_m}\right\}_{\max} = \begin{cases} \sqrt{1.425\dfrac{\varkappa}{k_1}}, \\ \sqrt{1.755\dfrac{\varkappa}{k_1}}, \\ \sqrt{1.5\dfrac{\varkappa}{k_1}}, \end{cases} \tag{14}$$

with $\varepsilon_t = (1/2\sigma^2)x(u_e - u_m)$

$$\left\{\frac{\sqrt{\overline{q^2}}}{u_e - u_m}\right\}_{\max} = \sqrt{\frac{1}{\pi}\,\frac{1}{2\sigma}}, \tag{15}$$

where it is assumed that $\varkappa = 0.011$, $k_1 = 0.4$, and $\sigma = 12$. For flow at a plate the total intensity maximum in a cross section of the viscous layer can be determined by Laufer's approximate equation [7]

$$\left(\frac{\sqrt{\overline{q^2}}}{u_e}\right)_{\max} = 2.2\sqrt{c_f}. \tag{16}$$

6. Critical Reynolds Number

Equations (14)-(16) are used to estimate the transition from laminar to turbulent motion. In accordance with the hypothesis advanced by L. N. Shchukin (see the above footnote), the transition occurs in that section of the boundary layer where the energy maximum of the neutral oscillations in the laminar layer coincides with the energy maximum of the pulsations in the developed turbulent flow (in this approach the length of the transition region is taken as equal to zero). In order to determine the time of the transition of the energy maximum we compared the energies of the neutral oscillation pulsations having a fixed E_0 and different ξ with the maximum total intensity calculated for flows of the wake type and for a separation point at the surface of the body based on Eqs. (14)-(15) with $u_m = 0$, and for flow along a plate from Eq. (16), where the coefficient of friction c_f was estimated from the well-known equation of L. V. Kozlov $c_f = 0.085\mathrm{Re}_x^{-0.29+0.01\log \mathrm{Re}_x}$, the Prandtl equation $c_f = 0.074\mathrm{Re}_x^{-0.2}$, the Schlichting – Prandtl equation $c_f = 0.455(\log \mathrm{Re}_x)^{-2.58}$, and the Folkner power-law equation $c_f = 0.0262\mathrm{Re}_x^{-1/7}$.

The results of the calculations are presented in Fig. 4, in which we show the generalized dependence of the transition Reynolds number ξ_{tr} on the degree of turbulence E_0 of the outer stream for the types of flows investigated. Good agreement with the experimental studies of Dryden is found for the plate ($\mathrm{Re}_{cr} \simeq 2.8\cdot 10^6$ with $E_0 = 0.0002$) [3]; the critical Reynolds number $2.34\cdot 10^6 \le (u_e x/\nu)_{cr} \le 2.82\cdot 10^6$ (the right value corresponds to the equation of L. V. Kozlov and the left, to the Prandtl equation) is obtained by calculation for the means of determination of c_f enumerated. The critical Reynolds number decreases sharply with an increase in the degree of turbulence E_0 in the outer stream. Dryden [3] pointed out this fact, noting that when $E_0 \ge 0.0003$ the transition to the turbulent form of flow is caused by random disturbances without a preliminary rise in the oscillation amplitude, as is assumed in the present analysis.

NOTATION

x, y, t, space – time coordinate system; u, v, longitudinal and transverse velocity components; $\Psi(x, y)$, $\psi'(x, y, t)$, stream functions of instantaneous component of primary flow and of perturbed motion, respectively; $\xi = (u_e x/\nu)$, current Reynolds number; $\beta = 2m/(m+1)$, parameter of pressure gradient $u_e = cx^m$ in Folkner – Scan flow; $\eta = y/x[\xi/(2-\beta)]^{0.5}$, similarity coordinate; $F(\eta)$, reduced dimensionless stream function; $F_\eta = u/u_e$, dimensionless longitudinal velocity; $\Phi'(t)$, a random function of time; $f(\eta)$, dimensionless pulsation function; $E_0 = \sqrt{\overline{u'^2}}/u_e$, degree of homogeneous turbulence in outer stream; Δ, Laplace operator; ν, kinematic viscosity; ρ, mass density of medium; $\overline{\Phi'^2}$, $\overline{u'^2}$, $\overline{v'^2}$, time averages of squares of pulsation components Φ', u', and v'; $\overline{q^2} = \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$, total pulsation intensity; $\tau = -\rho\overline{u'v'}$, Reynolds stress; ε_t, coefficient of turbulent viscosity; δ, thickness of boundary layer; $k_1 = 0.4$, proportionality constant (an experimental value) in the correlation function $\overline{u'v'} = \pm \overline{k_1 q^2}$; $\varkappa = 0.011$, turbulence constant in Prandtl equation for jet flows; $\sigma = 12$, similarity coordinate for turbulent mixing. Indices: η, x, y, t, derivative of the respective function with respect to the coordinates η, x, y, and t; e, outer boundary of the viscous layer; i = 0, 1, 2, 3, and 4, numbers of the coefficients α_i.

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APPROXIMATE METHOD OF CALCULATING THE TEMPERATURE PROFILE IN A SEMITRANSPARENT MELTING MATERIAL

A. S. Senchenkov

UDC 536.12:536.35

The integral two-parametric method is used to calculate the temperature profile in a semitransparent melting material; the method takes the specifics of the problem under study into account.

The velocity for the removal of the mass of vitreous heatproof materials under heating action is determined to a significant degree by the temperature profile near the surface, a fact which is related to the strong dependence of the viscosity of these materials on the temperature. Since most vitreous heatproof materials are semitransparent, the temperature profile also determines the amount of heat emitted by the material.

At the same time, as the results of the numerical calculations show [1], the exponential approximations of a transparent film and an opaque film and the approximation of radiant thermal conductivity do not guarantee the satisfactory accuracy in calculating the temperature distribution near the surface if the optic thickness of the liquid film has the order of unity (typical for many heatproof materials).

Below we propose an approximate method for calculating the temperature distribution in a semitransparent material that is applicable for the case given.

The fracture of heatproof materials under heating is described by a system of equations of continuity, motion, energy, and emission transfer with corresponding boundary conditions [2]. We limit ourselves in the present study to the energy equations. We can write the equation for a stationary regime of fracture in dimensionless form [1,2]

$$\frac{d}{dy}\left(\frac{d\theta}{dy} + \theta - f\right) = 0, \tag{1}$$

$$\theta(0) = 1, \qquad \theta(\infty) = \theta_{\mathrm{T}}.$$

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